## PROBLEM 2.38

A process instrument reading, $Z$ (volts), is thought to be related to a process stream flow rate $V$ (L/s) and pressure $P$ (kPa) by the following expression:

$$Z = \boldsymbol{a}\dot{V}^{\boldsymbol{b}} P^{\boldsymbol{c}}$$

Process data have been obtained in two sets of runs — one with $\dot{V}$ held constant, the other with $P$ held constant. The data are as follows:

| Point | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\dot{V}$ (L/s) | 0.65 | 1.02 | 1.75 | 3.43 | 1.02 | 1.02 | 1.02 |
| $P$ (kPa) | 11.2 | 11.2 | 11.2 | 11.2 | 9.1 | 7.6 | 5.4 |
| $Z$ (volts) | 2.27 | 2.58 | 3.72 | 5.21 | 3.50 | 4.19 | 5.89 |

**(a)** Suppose you had only performed Runs 2, 3, and 5. Calculate ***a***, ***b***, and ***c*** algebraically from the data for these three runs.

### Strategy

Everything is easier when you work with linear equations, so a natural strategy when you are dealing with exponential or power-law functions is to take natural logarithms. Since powers are involved in the given expression for $Z$, take the natural logarithm of both sides and then substitute values of $\dot{V}$ , $P$, $Z$ for the three data points. You will then get three equations in three unknowns (the three coefficients ***a***, ***b***, and ***c*** which we've bolded for clarity).

$$Z = \boldsymbol{a}V^{\mathbf{b}}P^{\mathbf{c}} \Rightarrow \ \ln Z = \ln\left(\boldsymbol{a}\right) + \boldsymbol{b}\ln\dot{V} + \boldsymbol{c}\ln P$$

using point (2): ln(2.58) = ln(***a***) + ***b*** ln (1.02) + ***c*** ln (11.2)
using point (3): ln(3.72) = ln(***a***) + ***b*** ln (1.75) + ***c*** ln (11.2)
using point (5): ln(3.5) = ln(***a***) + ***b*** ln (1.02) + ***c*** ln (9.1)

You can either solve the equations the hard way or the easy way. The hard way is to subtract (2) from (5) and find ***c,*** then subtract (2) from (3) and find ***b***, then substitute for ***b*** and ***c*** in any one of the equations to find ln(***a***), then find ***a*** as $e^{\ln(\boldsymbol{a})}$. The easy way is to enter the three equations into E-Z Solve (included in the CD-ROM that came with the text) and solve them with a single mouse click. We recommend doing it the easy way partly because it's the easy way and partly because if you get practice with E-Z Solve on simple problems such as this one, you'll be ready to tackle more complex problems when you get to them (which you will). Bring up E-Z Solve, enter the following program (first filling in the missing values), choose "Solve/Sweep" under the "Solutions" menu (or just type the F5 key on your keyboard), and click on "Solve" in the dialog that appears.

**E-Z Solve Code** **(2.38-1)**

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// Problem 2.38(a)
ln(2.58) = lna + b*ln(1.02) + c*ln(11.2)          // "lna" is a variable name & represents ln(a)
ln(3.72) = lna + b*ln(_____) + c*ln(_____)
ln(____) = lna + b*ln(_____) + c*ln(_____)
a = exp(lna)
```